Balancing Defending and Attacking—Both Governments’ and Terrorists’ Problem\(^1\)

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**Abstract**

We analyze how a government allocates resources between defending against a terrorist attack and attacking a terrorist’s resource, and how a terrorist allocates resources between attacking a government’s asset and defending its resource. A two-stage game is considered where the government moves first. When the terrorist’s resource is small, the government attacks the terrorist’s resource to deter the terrorist. When the terrorist’s resource is intermediate, the terrorist attacks and defends, while the government only attacks. When the terrorist’s resource is high, both players defend and attack. We analyze \(T\) periods of the two stage game. After an

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attack the government may enjoy incoming resources which deter the terrorist for a certain amount of periods. The terrorist’s resource may increase due to incoming funds. With arithmetic increase, the terrorist is deterred for several periods, after which the terrorist’s attack increases with slight convexity. With geometric increase, the terrorist is deterred through more periods, after which the terrorist’s attack increases with strong convexity.

**Nomenclature**

- \( T \): number of time periods
- \( t \): time period, \( t=1,\ldots,T \)
- \( r_t \): government’s resource in period \( t \), \( r_t \geq 0 \)
- \( R_t \): terrorist’s resource in period \( t \), \( R_t \geq 0 \)
- \( d_t \): government's defense effort protecting the asset in period \( t \), \( d_t \geq 0 \)
- \( A_t \): terrorist’s attack effort attacking the asset in period \( t \), \( A_t \geq 0 \)
- \( a_t \): government's attack effort attacking the terrorist’s resource \( R_t \) in period \( t \), \( a_t \geq 0 \)
- \( D_t \): terrorist’s defense effort protecting its resource in period \( t \), \( D_t \geq 0 \)
- \( c_t \): beginning security level in period \( t \) before any defensive investment is spent, \( c_t \geq 0 \)
- \( P_t \): probability of asset damage in period \( t \), \( 1 \geq P_t \geq 0 \)
- \( Q_t \): fraction of surviving terrorist resources in government’s attack, \( 1 \geq Q_t \geq 0 \)
- \( b_t \): government’s unit defense cost in period \( t \), \( b_t \geq 0 \)
- \( B_t \): terrorist’s unit attack cost in period \( t \), \( B_t \geq 0 \)
- \( g_t \): government’s unit attack cost in period \( t \), \( g_t \geq 0 \)
- \( G_t \): terrorist’s unit defense cost in period \( t \), \( G_t \geq 0 \)
- \( v_t \): government’s asset valuation, \( v_t \geq 0 \)
- \( V_t \): terrorist’s asset valuation, \( V_t \geq 0 \)
- \( m \): contest intensity for asset damage, \( m \geq 0 \)
- \( u_t \): government’s expected utility in period \( t \)
- \( U_t \): terrorist’s expected utility in period \( t \)

**1 Introduction**

Essential for terrorism assessment is the understanding of the terrorist’s and government’s objectives, resource capacities, and the role of time. Extending earlier research which has
typically assumed that the government defends and the terrorist attacks, this paper more realistically and ambitiously assumes that both players both defend and attack. The government defends its assets and infrastructures, but may also attack the terrorist’s resource base. Consequently, the terrorist defends its resource base in addition to attacking the government’s assets. We furthermore consider how the government and terrorist defend and attack through time. We analyze how the government may deter attacks, and how the terrorist responds to such deterrence. We model objectives as utilities, distinguish between unit costs of defense and attack, allow different asset valuations for the government and terrorist, and allow for a beginning security level (Zhuang and Bier, 2007). The interplay of these factors causes a variety of different equilibrium strategies which are analyzed.

We develop a model for how a government allocates resources between defending against a terrorist attack and attacking a terrorist’s resource base, and how a terrorist analogously allocates resources between attacking a government’s asset and defending its resource base. We consider the government and terrorist as unitary players. The government builds the defense of infrastructures over time. We assume that the terrorist takes this defense information as given when choosing its attack strategy in each time period. Accordingly, in each period, we analyze a two-stage game where the government moves in the first stage, and the terrorist moves in the second stage. Such a game is usually more descriptive than a simultaneous game where the players are unaware of each other’s actions For example, the U.S. homeland security defense budget and its Iraq and Afghanistan operations are well observed by the terrorist.

The two-stage game is played T times referred to as periods. The time between periods is assumed to be sufficiently longer than the time between stages so that each two-stage game can be solved with backward induction for each period. This means that the players are myopic and boundedly rational in the sense that they only consider one two-stage game in each period.

Clausewitz (1832) suggested that attack is the best defense. The principle is much discussed and does not always hold. This paper seeks to determine to what extent it is optimal to stay on the defensive and await the terrorist attack, and to what extent it is optimal to go on the offensive and actively decrease the terrorist’s resource. Both players have fixed resources which can be used defensively or offensively.
To facilitate analytical tractability of the attack versus defense balance for two players, accounting for the time factor, one asset is considered. The asset is interpreted broadly as something of value which the government seeks to protect and the terrorist seeks to destroy or capture. Some terrorists have a broad objective such as inflicting damage on a country as such, e.g. the United States. In face of such a terrorist, a government defends its entire country, which calls for a broad defense. The model also applies for collections of assets interpreted as a joint asset, and assets defined more narrowly, to the extent both the government and the terrorist can be perceived as allocating budgets for attack and defense by collections of assets or specific assets. One example of a collection of assets is the four targets of the 9/11 attack, i.e. the World Trade Center's North and South Towers, the Pentagon, and the White House (which was not hit). Focusing on one asset means that we do not analyze how the government and terrorist substitutes resources across assets. For that research question see Enders and Sandler (2003) and Hausken (2006), and Bier et al. (2007) for when a government allocates defense to a collection of locations while a terrorist chooses a location to attack.

Section 2 presents a literature review. Section 3 develops the model. The government allocates its resource into defending its asset and attacking the terrorist’s resource. The terrorist allocates the surviving part of its resource into attacking the government’s asset and defending its resource. The probability of asset damage, utilities, free choice variables, game structure, and equilibrium are specified. Section 4 analyzes the two-stage game and determines three cases for the solution; the terrorist is deterred, the government attacks but does not defend, or both players defend and attack. Section 5 illustrates the solution with simulations. Section 6 considers the T period game. It is first illustrated how the three cases arise and which strategies are chosen in two subsequent time periods dependent on various sizes of the terrorist’s resource. Thereafter the impact of letting the government’s resource bounce back after an attack is analyzed, which determines how the terrorist can be deterred. Finally the terrorist’s resource is allowed to increase arithmetically and geometrically, and the impact on deterring the terrorist from attacks is analyzed. Section 7 suggests how to validate the model and results. Section 8 concludes.

2 Literature review
To position the current paper within the stream of literature, we briefly outline earlier research. Much research has considered passive defense in the sense of defending against incoming attacks. Azaiez and Bier (2007) consider the optimal resource allocation for security in reliability systems. They determine closed-form results for moderately general systems, assuming that the cost of an attack against any given component increases linearly in the amount of defensive investment in that component. Bier et al. (2005) and Bier and Abhichandani (2002) assume that the government minimizes the success probability and expected damage of an attack. Bier et al. (2005) analyze the protection of series and parallel systems with components of different values. They specify optimal defenses against intentional threats to system reliability, focusing on the tradeoff between investment cost and security. The optimal defense allocation depends on the structure of the system, the cost-effectiveness of infrastructure protection investments, and the adversary's goals and constraints. Bier and Abhichandani (2002) apply game theory to characterize optimal defensive strategies against intentional attacks. Levitin (2007) considers the optimal element separation and protection in complex multi-state series-parallel system and suggests an algorithm for determining the expected damage caused by a strategic terrorist. Patterson and Apostolakis (2007) introduced importance measures for ranking the system elements in complex systems exposed to terrorist actions. Michaud and Apostolakis (2006) analyzed such measures of damage caused by the terror as impact on people, impact on environment, impact on public image etc. Dighe et al. (2009) consider secrecy in defensive allocations as a strategy for achieving more cost-effective terrorist deterrence. Zhuang and Bier (2007) consider government resource allocation for countering terrorism and natural disasters.

Hausken, Bier, and Zhuang (2009) consider a defender which chooses tradeoffs between investments in protection against natural disaster only, protection against terrorism only, and all-hazards protection, allowing sequential or simultaneous moves. Similarly, Zhuang and Bier (2007) study the balance between natural disaster and terrorism, where either the defender moves first (and the attacker second), or they move simultaneously. Levitin and Hausken (2008) consider a two period model where the defender, moving first, distributes its resource between deploying redundant elements and protecting them from attacks.
For a recent survey of work that examines the strategic dynamics of governments vs. terrorists, see Sandler and Siqueira (2009). They survey advances in game-theoretic analyses of terrorism, such as proactive versus defensive countermeasures, the impact of domestic politics, the interaction between political and militant factions within terrorist groups, and fixed budgets. Further, Brown et al. (2006) consider defender-attacker-defender models. First the defender first invests in protecting the infrastructure, subject to a budget constraint. Then, a resource-constrained attack is carried out. Finally, the defender operates the residual system as best possible. They exemplify with border control, the U.S. strategic petroleum reserve, and electric power grids.

Some research has focused on investment substitutions across time. First, Enders and Sandler (2003) suggest that a terrorist may compile and accumulate resources during times when the government’s investments are high, awaiting times when the government may relax his efforts and choose lower investments. Second, Keohane and Zeckhauser (2003:201,224) show that “the optimal control of terror stocks will rely on both ongoing abatement and periodic cleanup” of “a terrorist’s ‘stock of terror capacity’”. Enders and Sandler (2005) use time series to show that little has changed in overall terrorism incidents before and after 9/11. Using 9/11 as a break date, they find that logistically complex hostage-taking events have fallen as a proportion of all events, while logistically simple, but deadly, bombings have increased as a proportion of deadly incidents. Bandyopadhyay and Sandler (2008) consider the interaction between preemption and defense. For example, high-cost defenders may rely on preemption, while too little preemption may give rise to subsequent excessive defense.

Raczynski (2004) simulates the dynamic interactions between terror and anti-terror groups. Feichtinger and Novak (2008) use differential game theory to study the intertemporal strategic interactions of Western governments and terror organizations. They illustrate long-run persistent oscillations. Berman and Gavious (2007) study a leader follower game where the State provides counter terrorism support across multiple metropolitan areas to minimize losses, while the Terrorist attacks one of the metropolitan areas to maximize his utility. Berrebi and Lakdawalla (2007) consider for 1949-2004 how terrorists seek targets in Israel, responding to costs and benefits, and find that long periods without an attack signal lower risk for most localities, but higher risk for important areas. Barros et al. (2006) apply parametric and semiparametric hazard
model specifications to study durations between Euskadi Ta Askatasuna’s (ETA, a Spain-based terrorist group) terrorist attacks which seem to increase in summer and decrease with respect to e.g. deterrence and political variables. Udwadia et al. (2006) consider the dynamic behavior of terrorists, those susceptible to terrorist and pacifist propaganda, military/police intervention to reduce the terrorist population, and nonviolent, persuasive intervention to influence those susceptible to becoming pacifists. Hausken (2008) considers a terrorist which defends an asset which grows from the first to the second period. The terrorist seeks to eliminate the asset optimally across the two periods. Telesca and Lovallo (2006) find that a terror event is not independent from the time elapsed since the previous event, except for severe attacks which approach a Poisson process. This latter finding suggests that attack and defense decisions are not unit-periodic in nature, but that there are linkages through time. One objective of the current paper is to understand more thoroughly the nature of such linkages through time, affected by changes in resources, unit costs of defense and attack, etc.

Our paper builds upon and extends earlier research. On the one hand we enrich the one period model by allowing both the government and terrorist to both defend and attack. The government defends itself and at the same time attacks the terrorist’s resource. Analogously, the terrorist defends its resource and at the same time uses its surviving resource to attack the government. On the other hand we repeat the one period model T times to understand how long the terrorist can be deterred. A resourceful terrorist is obviously harder to deter than a less resourceful terrorist, but it is not obvious how a government should allocate its resource into defending itself and attacking the terrorist, and analogously how the terrorist should allocate its resource into defending itself and attacking the government. This paper seeks to determine the key factors that impact such resource allocations, which are important to understand for governments, policy makers, and terrorists.

3 The model
3.1 Motivation
The model in this paper seeks to answer the research question of how two players, a government and a terrorist, strike a balance between attack and defense through time. Game theory is chosen as the modeling methodology to account for the two players’ strategic options. Important factors related to this research question are the players’ resources, asset valuations, the contest
intensity for asset damage, the government’s beginning security level in each time period, and unit costs of defense and attack.

3.2 Assumptions
In each time period \( t, t=1,2,\ldots \), the government moves first by transforming the resource \( r_t \) to either defense \( d_t \) at unit cost \( b_t \), or attack \( a_t \) at unit cost \( g_t \) directed against the terrorist’s resource. The resource \( r_t \) can be a capital good, or labor. More specifically, using Hirshleifer’s (1995:30), Skaperdas and Syropoulos’ (1997:102), and Hausken’s (2005:62) terminology, \( b_t \) and \( g_t \) are unit conversion costs of transforming the resource \( r_t \) into \( d_t \) and \( a_t \), respectively, that is, \( r_t = b_t d_t + g_t a_t \) (1)

The transformation into \( d_t \) and \( a_t \) can be considered as production processes where \( 1/b_t \) and \( 1/g_t \) are productive efficiencies. Note that (1) implicitly requires that \( d_t \leq r_t \) and \( a_t \leq r_t \). Note also that allocating equal amounts of resources (e.g. a capital good such as money) into defense and attack (\( r_t/2 \) to each) generally does not mean that the defense effort \( d_t \) and attack effort \( a_t \) become equally large since the productive efficiencies of these two kinds of efforts may be different. For example, economies of scale, differences in competence and organizational structure, and different production processes, may cause \( 1/b_t \) and \( 1/g_t \) to differ substantially. However, if \( 1/b_t=1/g_t \) and \( r_t/2 \) is allocated to each kind of effort, then we have \( d_t=a_t \).

An allocation of a fixed and exogenously given resource into two kinds of efforts has earlier been made by Hirshleifer (1995) and Hausken (2005). A novelty in this paper is that \( r_t \) is exogenously given in each time period. Neither the defender nor the attacker affects \( r_t \) over time, but \( r_t \) may change over time due to external factors not endogenized in this paper. Endogenizing such external factors is challenging and may be done in future research.

We define \( r_t \) as the total resource available for the government in each time period. For example, if the government purchases and injects airplanes in one period, the airplanes may still be available in the next period, but the airplanes need maintenance, operating resources, personnel, etc., which are included in the total resource for the next period. The government’s resource can be perceived as allocated through a budget in each time period. Equation (1) can be
considered as striking a balance between defending an asset in one’s homeland against attack, on the one hand, and actively attacking and decreasing the terrorist’s resource, wherever it is located, on the other hand. Both governments and terrorists may to some extent have separate power fractions and decentralized decision making. For example, in the U.S. terrorism defense is to some extent separated in chain of command and funding channels from attack activities. However, moving towards the top of the chain of command, which in the U.S. means Congress and the president, resource allocation inevitably occurs between defense and attack. We thus consider the government and terrorist as unitary players. Multiple terrorist threats generated by one or multiple terrorists are either perceived as independent, or, if they have commonalities, they can be grouped together as a large threat generated by a collective actor, applying Simon’s (1969) principle of “near decomposability”. Future research may model the government and terrorist as non-unitary heterogeneous players.

The terrorist observes the government’s choice in each time period and allocates resources. We model the probability of damage of the terrorist’s resource with the common ratio form (Tullock 1980, Skaperdasas 1996) contest success function, i.e.

\[ Q_t(a_t, D_t) = \frac{D_t}{D_t + a_t} \]  

(2)

where \( \partial Q_t / \partial D_t > 0 \) and \( \partial Q_t / \partial a_t < 0 \). The terrorist’s original resource in each period is \( R_t \), but it decreases to \( Q_t R_t \) due to the government’s attack. The remaining resources \( Q_t R_t \) are transformed at unit cost \( B_t \) into attack \( A_t \) of an asset controlled by the government, or transformed at unit cost \( G_t \) into defense \( D_t \) against the government’s attack. The terrorist’s resource allocation equation can thus be expressed as

\[ Q_t R_t = \frac{D_t}{D_t + a_t} R_t = G_t D_t + B_t A_t \]  

(3)

Analogously to (1), \( R_t \) can be a capital good and labor, and \( G_t \) and \( B_t \) are unit conversion costs of transforming \( R_t \) into \( D_t \) and \( A_t \), respectively. Hence \( 1/G_t \) and \( 1/B_t \) are productive efficiencies. Equation (3) expresses that the terrorist needs to protect its entire resource. For example, when launching an attack, the terrorist needs to protect equipment and personnel involved in the attack. Equation (3) implies that there is an immediate feedback between the government’s attack and the terrorist’s allocation of \( R_t \) into \( D_t \) and \( A_t \). Prior to period \( t \), the terrorist
possesses the resource $R_t$. The defender moves first in a two-stage game in period $t$, and the attacker moves second. This means that the attacker possesses only $Q_t R_t$ when making its allocation in period $t$. The terrorist cannot allocate its fraction $(1-Q_t) R_t$ into defense and attack in period $t$ since that fraction has already been eliminated by the government in the first stage of period $t$.

For the probability of asset damage, we consider the following form of the contest success function,

$$P_t(d_t, A_t) = \frac{A_t^m}{A_t^m + (d_t + c_t)^m}$$

where $m \geq 0$ is a parameter for the contest intensity, $\partial P_t / \partial d_t < 0$ and $\partial P_t / \partial A_t > 0$. We have included $c_t$ which is the beginning security level at the beginning of period $t$ before any defensive investment is spent, which may depend on the histories of the players' strategies $\{d_{t-1}, d_{t-2}, \ldots\}$ and $\{A_{t-1}, A_{t-2}, \ldots\}$, and damage $\{P_{t-1}, P_{t-2}, \ldots\}$. See Amegashie (2006) and Zhuang and Bier (2007:981,983) for formulations akin to (4).

When $m=0$, the efforts $d_t$ and $A_t$ have no impact on the damage regardless of their size which gives 50% damage. When $0<m<1$, exerting more effort than one’s opponent gives less advantage in terms of asset damage than the proportionality of the players’ efforts specify. For example, $A_t=2$, $d_t+c_t=1$, $m=0.5$ gives $P_t=0.59 < 2/3$. When $m=1$, the efforts have proportional impact on the damage. When $m>1$, exerting more effort than one’s opponent gives more advantage in terms of vulnerability than the proportionality of the agents’ efforts specify. For example, $A_t=2$, $d_t+c_t=1$, $m=2$ gives $P_t=0.8 > 2/3$. Finally, $m=\infty$ gives a step function where “winner-takes-all”. The parameter $m$ is a characteristic of the contest which can be illustrated by the history of warfare. Low intensity occurs for assets which are defendable, predictable, and consisting of individual asset components which are dispersed, i.e. physically distant or separated by barriers of various kinds. Neither the government nor the terrorist can get a significant upper hand. An example is the time prior to the emergence of cannons and modern

\[\text{We consider } d_t \text{ and } c_t \text{ as operating jointly to constitute the government’s defense, and thus consider } (d_t + c_t)^m, \text{ instead of assuming that } d_t \text{ and } c_t \text{ operate separately, as if they were generated by different actors, which would give } d_t^m + c_t^m.\]
fortifications in the fifteenth century. Another example is entrenchment combined with the machine gun, in multiple dispersed locations, in World War I (Hirshleifer 1995:32-33). High m occurs for assets which are less predictable, easier to attack, and where the individual asset components are concentrated, i.e. close to each other or not separated by particular barriers. This may cause “winner-take-all” battles and dictatorship by the strongest. Either the government or the terrorist may get the upper hand. The combination of airplanes, tanks, and mechanized infantry in World War II allowed both the offense and defense to concentrate firepower more rapidly, which intensified the effect of force superiority.

When \( m>1 \), exerting more effort than one’s opponent gives more advantage in terms of vulnerability than the proportionality of the agents’ efforts specify. For example, \( T=2, t=1, m=2 \) gives \( v=0.8 > 2/3 \). Finally, \( m=\infty \) gives a step function where “winner-takes-all”. The parameter \( m \) can be illustrated by the history of warfare. Low intensity occurs in situations where neither the defender nor the attacker can get a significant upper hand. Examples are the time prior to cannons and modern fortifications in the fifteenth century, and entrenchment used with the machine gun in World War I (Hirshleifer 1995:32-33). High m occurs when one or the other opponent more easily can get the upper hand. Airplanes, tanks, and mechanized infantry in World War II allowed both the offense and defense to concentrate firepower more rapidly, which intensified the effect of force superiority.

A good model should have neither too many nor too few parameters. With too few parameters, the essence of reality is not captured. With too many parameters, distinguishing the different causal relationships becomes difficult. The current model strikes a balance between these opposing concerns by choosing 10 parameters. Four parameters are unit costs. Four parameters are the two players’ resources and asset valuations. The two final parameters are the beginning security level \( c_t \) and the contest intensity for asset damage \( m \).

### 3.3 Problem formulation

The probability that the asset is not damaged is \( P(d_A, A_t) \), which the government maximizes, accounting for the asset valuation \( v_t \). Analogously, the terrorist maximizes the damage
accounting for the asset valuation $V_t$. The government's and terrorist's expected utilities in period $t$ are

$$u_t(d_t, A_t) = [1 - P_t(d_t, A_t)]v_t = \frac{(d_t + c_t)^m v_t}{A_t^m + (d_t + c_t)^m}$$

$$U_t(d_t, A_t) = P_t(d_t, A_t)V_t = \frac{A_t^m V_t}{A_t^m + (d_t + c_t)^m}$$

Inserting (1) and (3) into (5) gives

$$u_t(a_t, D_t) = \left( \frac{r_t - g_t a_t}{b_t} + c_t \right)^m v_t$$

$$U_t(a_t, D_t) = \left( D_t \left[ \frac{R_t}{D_t + a_t} - G_t \right] / B_t \right)^m + \left( \frac{r_t - g_t a_t}{b_t} + c_t \right)^m$$

The government’s one free choice variable is $a_t$, where $d_t$ follows from (1). Analogously, the terrorist’s one free choice variable is $D_t$, where $A_t$ follows from (3). We assume common knowledge so that both players know all parameters and the game structure.

In each time period $t$ we consider a two-stage game where the government moves first, and the terrorist moves second. To determine the Subgame Perfect Nash Equilibrium (see Chapter 9.B in Mas-Colell et al. 1995) we assume that the government chooses $a_t$ in the first stage. The terrorist observes $a_t$ and chooses $D_t$ in the second stage. The game is solved with backward induction.

**Definition 1.** A strategy pair $(a^S_t, D^S_t)$ is a Subgame Perfect Nash Equilibrium if and only if

$$D^S_t = D_t(a^S_t) = \arg \max_{D_t \geq 0} U_t(a^S_t, D_t)$$

and

$$a^S_t = \arg \max_{a_t \geq 0} u_t(a_t, D_t(a_t))$$
4 Solving the two-stage game

Solving the game with backward induction, the Appendix determines the interior solution

\[ a_i = \frac{G_i(r_i + b_i c_i)^2}{g_i^2 R_i}, \quad d_i = \frac{1}{b_i} \left( r_i - \frac{G_i(r_i + b_i c_i)^2}{g_i R_i} \right) \]  

(9)

\[ D_i = \frac{(r_i + b_i c_i)}{g_i} \left( 1 - \frac{G_i(r_i + b_i c_i)}{g_i R_i} \right), \quad A_i = \frac{R_i}{B_i} \left( 1 - \frac{G_i(r_i + b_i c_i)}{g_i R_i} \right)^2 \]  

(10)

Both players’ choice variables are positive when

\[ R_i \geq \frac{G_i(r_i + b_i c_i)^2}{g_i r_i} \]  

(11)

When (11) is not satisfied, the government refrains from defending, \( d_i = 0 \), and focuses on attack, \( a_i = \frac{r_i}{g_i} \), as determined by (1). The Appendix shows that the solution is

\[ D_i = \sqrt{\frac{r_i}{g_i}} \left( \sqrt{\frac{R_i}{G_i}} - \sqrt{\frac{r_i}{g_i}} \right), \quad A_i = \frac{G_i}{B_i} \left( \sqrt{\frac{R_i}{G_i}} - \sqrt{\frac{r_i}{g_i}} \right)^2 \]  

(12)

Assuming that (10) does not hold and that \( D_i > 0 \) and \( A_i > 0 \) implies

\[ \frac{G_i r_i}{g_i} \leq R_i \leq \frac{G_i(r_i + b_i c_i)^2}{g_i r_i} \]  

(13)

which means that the government relies on \( c_i \) for the defense and the terrorist chooses \( D_i > 0 \) and \( A_i > 0 \). When \( c_i \) decreases toward 0, the range for \( R_i \) in (13) shrinks toward 0.

Equations (10) and (12) show that the terrorist jointly increases \( D_i \) and \( A_i \) above 0. The attacker either chooses \( D_i = A_i = 0 \) or \( \{D_i > 0, A_i > 0\} \). Intuition for this can also be gathered from (3). The terrorist’s defense effort \( D_i \) is positive if and only if its attack effort \( A_i \) is positive. This follows since the reason for the terrorist to defend is to ensure that resources are available to attack; and if the terrorist does not defend, then there are no resources available to attack. Conversely, as long as \( a_i \geq \frac{R_i}{G_i} \), the terrorist will give up defending and the government can eliminate the terrorist’s resource.
Equation (12) implies that the terrorist withdraws and chooses $D_t = A_t = 0$ when $R_t = G_t r_t / g_t$, which means that the terrorist is deterred by $c_t$ and the government’s $a_t = (r_t / g_t)$. When $R_t \leq G_t r_t / g_t$, less government deterrence is needed.

As $R_t$ decreases below $R_t = G_t r_t / g_t$, less government resources are needed to deter the terrorist. In particular, using (A1), we note that $a_t = R_t / G_t$ is sufficient to deter the terrorist. In this case, using (1) when $d_t = 0$, we assume that the government uses $r_t^* = g_t R_t / G_t \leq r_t$ which deters the terrorist and saves $r_t - r_t^*$ of resources. Table 1 summarizes the solutions for the Subgame Perfect Nash Equilibrium.

Table 1: Solution to Subgame Perfect Nash Equilibrium

<table>
<thead>
<tr>
<th>Cases</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditions</td>
<td>$R_t \leq G_t r_t / g_t$</td>
<td>$G_t r_t \leq R_t \leq (G_t r_t / g_t)^2$</td>
<td>$R_t \geq G_t (r_t + b c_t)^2 / g_t r_t^*$</td>
</tr>
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<td>Scenarios</td>
<td>Inactive terrorist; deterring government</td>
<td>Active terrorist; non-defending government</td>
<td>Active terrorist and government</td>
</tr>
<tr>
<td>$a_t$</td>
<td>$\frac{R_t}{G_t}$</td>
<td>$\frac{r_t}{g_t}$</td>
<td>$\frac{G_t (r_t + b c_t)^2}{g_t R_t}$</td>
</tr>
<tr>
<td>$d_t$</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{b_t} \left( r_t - \frac{G_t (r_t + b c_t)^2}{g_t R_t} \right)$</td>
</tr>
<tr>
<td>$A_t$</td>
<td>0</td>
<td>$\frac{G_t (r_t - G_t r_t / g_t)}{R_t}$</td>
<td>$\frac{G_t}{B_t} \left( 1 - \frac{G_t (r_t + b c_t)}{g_t R_t} \right)$</td>
</tr>
<tr>
<td>$D_t$</td>
<td>0</td>
<td>$\frac{r_t}{g_t} \left( \frac{R_t}{G_t} - \frac{r_t}{g_t} \right)$</td>
<td>$\left( r_t + b c_t \right) \left( \frac{1 - G_t (r_t + b c_t)}{g_t R_t} \right)$</td>
</tr>
<tr>
<td>$u_t$</td>
<td>$\nu_t$</td>
<td>$\frac{G_t}{R_t} \left( \frac{R_t}{G_t} - \frac{r_t}{g_t} \right) + c_t^n$</td>
<td>$\nu_t$</td>
</tr>
</tbody>
</table>

The table entries are calculated using the equations provided in the text.
There are three cases of solutions classified by the terrorist’s resource $R_t$:

Case 1: When the terrorist’s resource is small, the terrorist is deterred with $a_t>0$; there is no terrorist activity, and the government keeps the whole asset.

Case 2: When the terrorist’s resource is intermediate, the government relies on its beginning security $c_t$ for the defense $(d_t=0)$, and attacks with $a_t>0$. The terrorist attacks the government’s asset and defends its resource. With no beginning security level $(c_t=0)$, case 2 does not exist.

Case 3: When the terrorist’s resource is high, both the government and terrorist defend and attack.

One should be careful when providing examples to illustrate the three cases since assessments are needed as to whether the examples fit the modeling assumptions. A variety of lists exist for terrorist organizations.\(^3\) Case 1 suggests that governments may handle minor terrorist threats with proactive attacks against their modest resource base. One example is the U.S. President Reagan’s attack on Tripoli and Benghazi April 14, 1986, after which Libya disappeared from media attention as a sponsor of terrorist attacks. One example of a contradictory anecdote for case 1 is when attacking a terrorist with scarce resources causes hatred to emerge within this terrorist which draws resources so that this terrorist becomes a larger future threat. (This possibility is handled in section 6.3 where the terrorist’s resource is allowed to increase for a variety of reasons.) At the other extreme, case 3 is illustrated by Al-Qaeda which both attacks and defends, and faces governments which attack and defend at home and abroad. Taliban is a possible example of case 2 as viewed by the United States. It is attacked in, e.g., Afghanistan, but not defended specifically against in the United States aside from general defense against terrorism.

---

Table 1 shows that the government always attacks the terrorist’s resource, choosing \( a_t > 0 \), and does not defend in cases 1 and 2. This result is interesting, especially with the many minor terrorist threats around the world (assuming these are independent, or grouped together applying Simon’s (1969) principle of “near decomposability”). It is really in a government’s interest to eliminate these with active defense \( a_t > 0 \). We show that minor terrorists with \( R_t \leq G_t r_t / g_t \) are fully deterred in case 1. As a terrorist grows more resourceful from case 1 to case 3 in Table 1, acknowledging that the government’s resources are finite, the government needs to strike a balance between active defense \( a_t \) and passive defense \( d_t \) as in case 3. On the one hand, this decreases the terrorist’s resources available for attack and protects the asset against the terrorist’s attack furnished by the terrorist’s resources which have not been eliminated by \( a_t \). As the terrorist’s resource increases from case 1 to case 3 in Table 1, the government gradually suffers a more inferior position: when the terrorist resource is low, the government applies a small resource to destroy the terrorist’s resource; otherwise the government applies its entire resource striking a balance between active and passive defense.

In case 1 the contest intensity plays no role. In cases 2 and 3 the government’s and terrorist’s strategic choices are equally unaffected by the contest intensity \( m \), but their utilities are affected by \( m \). Case 2 implies

\[
\frac{\partial u_t}{\partial m} > 0 \Leftrightarrow \frac{B_t c_t}{G_t \left( \frac{R_t}{G_t} - \sqrt{\frac{r_t}{g_t}} \right)^2} > 1, \quad \frac{\partial U_t}{\partial m} > 0 \Leftrightarrow \frac{B_t c_t}{G_t \left( \frac{R_t}{G_t} - \sqrt{\frac{r_t}{g_t}} \right)^2} < 1
\]  \hspace{1cm} (14)

Case 3 implies

\[
\frac{\partial u_t}{\partial m} > 0 \Leftrightarrow \frac{1}{b_t} \left( \frac{r_t - G_t (r_t + b_t c_t)^2}{g_t R_t} \right)^2 + c_t > 1, \quad \frac{\partial U_t}{\partial m} > 0 \Leftrightarrow \frac{1}{b_t} \left( \frac{r_t - G_t (r_t + b_t c_t)^2}{g_t R_t} \right)^2 + c_t < 1
\]  \hspace{1cm} (15)

Consequently, changes in the contest intensity always benefits one actor and hurts the other.
Theorem 1. (a) When \( R_i \leq G_i r_i / g_i \), the terrorist is fully deterred with a government attack effort \( a_i = R_i / G_i \) and the government does not defend. (b) When \( \frac{G_i r_i}{g_i} \leq R_i \leq \frac{G_i (r_i + b_i c_i)^2}{g_i r_i} \), the government attacks with \( a_i = r_i / g_i \) does not defend, \( d_i = 0 \), while the terrorist attacks and defends. (c) When \( R_i \geq \frac{G_i (r_i + b_i c_i)^2}{g_i r_i} \), both players attack and defend.

Proof: Follows from Table 1.

5 Illustrating the two-stage game

To determine plausible parameter values we reason as follows. Both players may have a variety of production processes for their four kinds of efforts. An especially common and salient ceteris paribus starting point is to assume that all the four unit costs of effort are equal, and set to 1, that is \( b_t = B_t = g_t = G_t = 1 \). The most plausible value for the contest intensity is also \( m = 1 \), which means that the players’ efforts have proportional impact on the damage. The beginning security level can be chosen from 0 and upwards, but also here we choose \( c_t = 1 \) as a common benchmark. Our next plausible assumption is that the players have equal asset valuations \( v_t = V_t \), which impact utilities but not efforts. Hence \( v_t = V_t \) is a scaling issue where we choose \( v_t = V_t = 10 \). A further benchmark is that the players are equally resourceful, \( r_t = R_t \), though this latter assumption will be altered substantially as we proceed, through changing \( R_t \). Using (1) and (3), where unit costs are 1, we choose \( r_t = R_t > 1 \) to get conveniently sized efforts. We have analyzed the impact on the solution in section 4 and Table 1 of varying \( r_t = R_t \) upwards and downwards, and have found that \( r_t = R_t = 10 \) is a plausible benchmark.

This section illustrates the two-stage game with the baseline values \( c_t = b_t = B_t = g_t = G_t = m = 1 \), \( r_t = v_t = V_t = 10 \). Figure 1 sets \( R_t = 10 \) and shows the equilibrium four choice variables \( a_t \), \( d_t \), \( A_t \), \( D_t \) and the two utilities \( u_t \) and \( U_t \) as the parameter values \( r_t \), \( R_t \), \( g_t \), \( G_t \) respectively, change from the baseline value.
In the first panel, when \( r_t > 10 \) (case 1), the terrorist withdraws. When \( 7.87 < r_t < 10 \) (case 2), the government does not defend. When \( r_t < 7.87 \) (case 3) both players’ defenses are inverse U shaped in an interior solution. For high \( r_t \) the government defends modestly out of strength, instead relying on attack. For low \( r_t \) the government defends modestly out of weakness.

In the second panel, when \( R_t < 10 \) (case 1), the terrorist withdraws. When \( 10 < R_t < 12.1 \) (case 2), the government does not defend. \( R_t > 12.1 \) (case 3) gives the interior solution where, in accordance with the right column in Table 1, both players’ defenses and the terrorist's utility increase asymptotically toward constants as \( R_t \) reaches infinity, that is \( \lim_{R_t \to \infty} d_t = 10 \),
\[
\lim_{R_t \to \infty} D_t = 11, \quad \lim_{R_t \to \infty} U_t = 10, \quad \text{the terrorist’s attack increases toward infinity,} \quad \lim_{R_t \to \infty} A_t = +\infty, \quad \text{and}
\]
the government’s attack and utility decrease toward zero, \( \lim_{R_t \to \infty} a_t = 0 \), \( \lim_{R_t \to \infty} u_t = 0 \).

In the third panel, \( g_t < 1 \) (case 1) deters the terrorist. As \( g_t \) increases above 1, the government allocates less resources to attack and more to defense, while the terrorist allocates less resources to defend and more to attack when \( g_t > 2.2 \).

In the fourth panel, \( G_t > 1 \) (case 1) deters the terrorist, and the government decreases its attack resource according to \( R_t / G_t \) as \( G_t \) increases.

Figure 2 keeps the baseline in Figure 1 but sets \( R_t = 20 \), which makes the terrorist twice as resourceful, and shows the six equilibrium variables as the eight parameters change from their baseline values. The proportional impact of \( v_t \) and \( V_t \) on the utilities, and no impact on the choice variables, are obvious and not included.
Figure 2 Equilibrium behavior as functions of $r_t$, $R_t$, $g_t$, $G_t$, $m$, $b_t$, $B_t$, $v_t$, $V_t$, and $c_t$ using baseline value $R_t=20$

As shown in Figure 2, the equilibrium behavior as a function of $r_t$, $R_t$, $g_t$, and $G_t$ are similar to those in Figure 1. Increasing the contest intensity $m$ has no impact on the choice variables, but
increases the government’s utility and decreases the terrorist’s utility since, for case 3,
\[
\frac{1}{b_t} \left( r_t - \frac{G_i(r_t + b_t c_t)^2}{g_j R_t} \right) + c_t = 1.22 > 1
\]
and therefore according to (15) we have \( \frac{\partial u_t}{\partial m} > 0 \) and
\[
\frac{\partial U_t}{\partial m} < 0 .
\]
Increasing the government’s unit defense cost \( b_t \) causes the government to rely more
on attack instead of defense. By contrast, increasing the terrorist’s unit attack cost \( B_t \) causes the
terrorist to decrease the attack, without impact on his defense. The players’ asset valuations \( v_t \)
and \( V_t \) impact utilities but not choice variables. Finally, the beginning security level \( c_t \) benefits
the government allowing it to decrease its defense and thus has more resources to attack.

6 The T period game
This section considers the same baseline as in Figure 2, that is \( c_t=b_t=B_t=g_t=G_t=m=1, r_t=v_t=V_t=10 \), where \( R_t \) varies from 9 through 11 to 13.

6.1 A 2 period game example
This section repeats the two stage game \( T=2 \) times. We assume that the time between periods is
sufficiently longer than the time between stages so that each two stage game can be solved with
backward induction. This means that the players are myopic and boundedly rational and only
consider one two-stage game in each period. These assumptions are made for analytical
tractability and because of the nature of real world interactions. First, analyzing a \( T \) period
game with backward induction from period \( T \), and simultaneously analyzing the embedded
two stage game with backward induction from stage 2 in period \( t=1, \ldots, T \), is an
insurmountable task. Second, real-world players are indeed myopic and boundedly rational
and do not look too far ahead because of the plethora of eventualities and unforeseen
contingencies that may arise. Survival in the present is important, and there is a tendency to
discount events in the remote future unless they can be demonstrated to be important.
Additionally, politically elected officials are usually elected for limited amounts of time. We
thus assume that the government and the terrorist maximize \( u_t \) and \( U_t \), respectively, in each
period. Without parameter changes, the 2 period game gives the same solution in each period as
the one period game. The unit attack cost $B_t$ is present on both sides of all inequalities in Table 1 and conveniently illustrate the four cases. Similar examples can be set up by altering of $r_t$, $R_t$, $g_t$, $G_t$, $m$, $b_t$, $B_t$, $v_t$, $V_t$, and $c_t$. Table 2 shows six examples of the government’s and terrorist’s equilibrium strategies when the terrorist’s resource $R_t$ increases from 9 through 11 to 13.

**Table 2: A 2 period numerical example of Subgame Perfect Nash Equilibrium**

<table>
<thead>
<tr>
<th>Examples</th>
<th>Conditions</th>
<th>$a_1$</th>
<th>$d_1$</th>
<th>$A_1$</th>
<th>$D_1$</th>
<th>$a_2$</th>
<th>$d_2$</th>
<th>$A_2$</th>
<th>$D_2$</th>
<th>Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$R_1=9$, $R_2=9$</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1,1</td>
</tr>
<tr>
<td>2</td>
<td>$R_1=9$, $R_2=11$</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0.02</td>
<td>0.49</td>
<td>1,2</td>
</tr>
<tr>
<td>3</td>
<td>$R_1=9$, $R_2=13$</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9.31</td>
<td>0.69</td>
<td>0.31</td>
<td>1.69</td>
<td>1,3</td>
</tr>
<tr>
<td>4</td>
<td>$R_1=11$, $R_2=9$</td>
<td>10</td>
<td>0</td>
<td>0.02</td>
<td>0.49</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,1</td>
</tr>
<tr>
<td>5</td>
<td>$R_1=11$, $R_2=11$</td>
<td>10</td>
<td>0</td>
<td>0.02</td>
<td>0.49</td>
<td>10</td>
<td>0</td>
<td>0.02</td>
<td>0.49</td>
<td>2,2</td>
</tr>
<tr>
<td>6</td>
<td>$R_1=11$, $R_2=13$</td>
<td>10</td>
<td>0</td>
<td>0.02</td>
<td>0.49</td>
<td>9.31</td>
<td>0.69</td>
<td>0.31</td>
<td>1.69</td>
<td>2,3</td>
</tr>
<tr>
<td>7</td>
<td>$R_1=13$, $R_2=9$</td>
<td>9.31</td>
<td>0.69</td>
<td>0.31</td>
<td>1.69</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3,1</td>
</tr>
<tr>
<td>8</td>
<td>$R_1=13$, $R_2=11$</td>
<td>9.31</td>
<td>0.69</td>
<td>0.31</td>
<td>1.69</td>
<td>10</td>
<td>0</td>
<td>0.02</td>
<td>0.49</td>
<td>3,2</td>
</tr>
<tr>
<td>9</td>
<td>$R_1=13$, $R_2=13$</td>
<td>9.31</td>
<td>0.69</td>
<td>0.31</td>
<td>1.69</td>
<td>9.31</td>
<td>0.69</td>
<td>0.31</td>
<td>1.69</td>
<td>3,3</td>
</tr>
</tbody>
</table>

The rightmost column shows which case in Table 1 each example corresponds to for the two periods. From Table 2 we see the following: First, the result is not time dependent, since we assume no linkage between periods. Second, when the terrorist resources are low, $R_t<10$, case 1 arises and the terrorist is deterred in both periods. This gives no terrorist activity since all such resources are going to be destroyed by a government’s attack. Third, when the terrorist resources are intermediate, $10<R_t<12.1$, case 2 arises and the government attacks and relies on $c_t$ for the defense, while the terrorist attacks and defends. Finally, when the terrorist’s resource is high, $R_t>12.1$, case 3 arises and both the government and the terrorist attack and defend.

### 6.2 Modeling the government’s resource $r_t$ bouncing back after an attack

Assume that $R_t > G_r r_t / g_s$ so that an attack occurs in period $s$. Assume for simplicity that all other parameters remain fixed for all $t \geq s$, while the government’s resource $r_t$ changes. This is because after an attack in period $s$ the government more easily acquires funding from various sources. With a significantly higher $r_t$, the condition $R_t > G_r r_t / g_s$ is no longer satisfied, which deters the terrorist. Assume that $r_t$ follows the form for $t \geq s$: 

The rightmost column shows which case in Table 1 each example corresponds to for the two periods.
where $r_{\text{max}}$ is the maximum value that $r_t$ acquires in period $s+1$ after a terrorist attack $A_s$ in period $s$, and $\Phi > 0$ regulates how quickly $r_t$ bounces concavely back and decreases to its original level $r_s$ due to defense reinforcement. Equation (16) states that $r_t = r_s$ in period $s$, that $r_t = r_{\text{max}}$ in period $s+1$, and thereafter decreases. As discussed below (1), $r_t$ continues to be exogenously given in each time period. Modeling the political processes after a terrorist attack is challenging and left for future research. Since $\lim_{t \to \infty} r_t = r_s$ and $\partial r_t / \partial t < 0$, a second attack eventually occurs in period $t^*$ determined by the smallest $t$ such that $R_t > G r_t / g_t$ (see Table 1) which implies

$$
t^* = \left\lfloor s + 1 + \frac{1}{\Phi} \ln \left( \frac{r_{\text{max}} - r_s}{R_t g_t / G_t - r_s} \right) \right\rfloor \tag{17}
$$

where $\left\lfloor z \right\rfloor$ is the least integer that is not less than $z$. When $F=0$, $r_t = r_{\text{max}}(A_s)$ for all $t>s$ and the terrorist is always deterred. When $F=\infty$, $r_t = r_s$ for all $t>s$ and the attacker attacks in each subsequent period. Using the same parameter values as in section 5, aside from $r_t$, Figure 3 shows the six equilibrium values $u_t$, $U_t$, $a_t$, $d_t$, $A_t$, $D_t$, and $r_t$ as functions of time $t$ when $s=0$, $r_{\text{max}} = 40$, and $F=1$. The terrorist is deterred in periods 1 and 2 by the substantial government’s attack $a_t=20$ since $R_t < G r_t / g_t$, is satisfied causing case 1. In period $t^* = 3$, $r_t$ drops to 14 according to (16). Therefore $R_t < G r_t / g_t$ is no longer satisfied, and we get case 3, in which the terrorist resumes activities.
6.3 Modeling the terrorist’s resource $R_t$

This section keeps the government’s resource at its baseline $r_t = 10$ and assumes that the terrorist’s resource $R_t$ increases from a low level. First, the terrorist may be new on the market and may acquire increasing funding from various sources if its objective gains support. Second, the terrorist may be established on the market, may have depleted its resources in earlier attacks, and may, if earlier attacks were successful, more easily acquire further resources. For example, attacks such as 9/11 attack may over the subsequent time generate sufficient momentum and willing investors among supporters of the attack to furnish a high $R_t$. Third, the terrorist may experience hatred (Glaeser 2005) arising from a variety of sources, which may draw volunteers and funding, which may get directed at governments. We investigate the following two functional forms of increment: arithmetic $R_t = 1 + t$ and geometric $R_t = 1.097^t$ for $t = 1, \ldots, 40$. Note that in both cases $R_t$ increases from 1 to 41 over 40 periods. Figure 4 shows the equilibrium dynamics of those two scenarios.

Figure 3 Equilibrium behavior as a function of time period when $r_t$ is dynamic
With arithmetically increasing $R_t$, the terrorist is deterred by the government’s attack through period 8, and the government does not defend (case 1). Aided by the beginning security $a_t$, the government does not attack in periods 9-11, while the terrorist’s attack increases with slight convexity causing increasing terrorist utility and decreasing government utility (case 2). From period 12, both players engage in full activities (case 3). With geometrically increasing $R_t$, the terrorist is deterred through period 24 (case 1), since it then takes much more time to cause $R_t > G_t r_t / g_t$. In periods 25-27 there is no government’s defense (case 2). From period 28 (case 3) the terrorist’s attack increases with strong convexity.

7 Suggestions for how to validate the model and results

This paper has developed a model with intuitive reasoning, has solved the model with game theoretic tools, has presented three cases for the size of the terrorist’s resource, has accounted for changes through time, and has illustrated the solution for various parameter values. Future research should support the model empirically and validate the results. Parameters should be estimated and tuned to match real-world cases. Cases that have occurred are a natural starting point. Proceeding through the parameters and variables in the nomenclature list, the government’s resource, asset valuation, and unit costs of defense and attack are determined from public records, interviews, and estimation techniques. The asset valuation can be estimated by
letting people and elected officials rank the value of multiple assets against each other. The terrorist’s resource, asset valuation, and unit costs of defense and attack are attempted estimated analogously, applying covert techniques and espionage, exploring statements and interviewing defectors and sympathizers of potential attackers, and applying expert judgments. Methods common in decision theory may also be used to estimate the parameters experimentally.

Once a collection of real world cases have been compiled with estimated values of parameters and variables through time, the next step is to analyze whether these cases comply with the solution predicted in this paper. First we determine whether a small terrorist’s resource causes the government to attack it for deterrence purposes, whether an intermediate terrorist’s resource causes the terrorist to attack and defend, and whether a high terrorist’s resource causes both players to defend and attack. Second we estimate empirically how the parameters and especially the government’s resource alters after an attack, determine whether or how long the terrorist gets deterred, and whether the deterrence period matches the deterrence period predicted in this paper. Third we estimate empirically how the terrorist’s resource changes after an attack, whether arithmetic or geometric increase is descriptive, and whether the impact on the players’ strategies matches the solution predicted in this paper.

The solutions are discussed with policy administrators. Once some agreement has been reached on parameter values, one may proceed with cases that may occur with varying degrees of likelihood, and prescribe optimal policies for each case. Prior to the 9/11 attack the notion of flying airplanes into buildings had been contemplated by various professionals, but had been assessed as too speculative for serious consideration. There is a need to proceed through both likely and unlikely scenarios and assess the optimal government response for each scenario.

8 Conclusion
The paper assumes that a government allocates resources between defending against a terrorist attack and attacking a terrorist’s resource base. Analogously we assume that a terrorist allocates resources between attacking a government’s asset and defending its resource base. The government builds the defense of infrastructures over time. The terrorist takes this defense as given when choosing its attack strategy at each time period. In each period, we analyze a two-
stage game where the government moves in the first stage, and the terrorist moves in the second stage.

There are three cases of solutions classified by the terrorist’s resource. First, when the terrorist’s resource is small, the government attacks the terrorist’s resource base sufficiently to deter the terrorist from attacking, and does not defend. This interesting result suggests that governments may handle the many minor terrorist threats around the world with proactive attacks against their modest resource bases, rather than designing a passive defense to passively await these to grow large and require more substantial resources. Second, when the terrorist’s resource is intermediate, the government attacks and relies on its beginning security for the defense, while the terrorist attacks the government’s asset and defends its resource. Third, when the terrorist’s resource is high, both the government and terrorist defend and attack.

Repeating the two stage game, we first consider two periods and show how the three cases arise dependent on changes in the terrorist’s resource. Second we allow the government to bounce back after an attack, caused by an increase in its resource driven by easier access to funding, with subsequent gradual decrease to its initial level. We show how effectively the government deters attacks during this period of higher resource availability, and when the terrorist is no longer deterred from further attacks. This transition to renewed terrorist attacks occurs when the terrorist’s resource is again larger than the government’s resource multiplied with the ratio of the terrorist’s unit defense cost and the government’s unit attack cost. The intuition is that when the terrorist is resourceful and enjoys a low unit defense cost, while the government is not resourceful and has a high unit attack cost, then terrorist attacks resume. Third, we consider the impact of increasing the terrorist’s resource from a low level, because the terrorist is either unestablished and gaining increased funding or is established and gaining easier access to funding due to the success of earlier attacks. We consider arithmetic and geometric resource increase for the terrorist resources. With arithmetic increase, the terrorist is deterred for a moderate amount of periods, after which both players engage in both defense and attack as the terrorist’s attack increases with slight convexity causing increasing terrorist utility and decreasing government utility. With geometric increasing, the terrorist is deterred over more periods since the terrorist resource buildup is slower. As the resource buildup gains momentum, the terrorist’s attack increases with strong convexity.
The model in this paper intends to be useful for legislators, the military, and government leaders. Based on some plausible assumptions, the strategic nature of the interaction between a government and a terrorist is captured causing policy recommendations for when and to what extent the government should defend itself versus attacking the terrorist, and how the terrorist responds by either being deterred, attacking, or defending its resource. Striking a balance between realism and parsimony, the model accounts for the players’ resources, asset valuations, the contest intensity for asset damage, the government’s beginning security level in each time period, and unit costs of defense and attack. Future research may account for alternative factors. We have also assumed boundedly rational players where the time between periods is sufficiently longer than the time between stages so that each two-stage game can be solved with backward induction for each period. Future research may search for alternative ways of modeling the challenge of interaction through time.

Appendix

We solve the game with backward induction, starting with the second stage. For any given government’s attack $a_t$, maximizing the terrorist’s utility $U_t(a_t, D_t)$ specified in (6) gives the terrorist best-response function:

$$D_t(a_t) = \arg \max_{D_t \geq 0} U_t(a_t, D_t) = \begin{cases} 0 & \text{if } a_t \geq \frac{R_t}{G_t} \\ \sqrt{\frac{R_t a_t}{G_t}} - a_t & \text{if } a_t \leq \frac{R_t}{G_t} \end{cases} \quad (A1)$$

From (A1) we see that the optimal terrorist’s defense level $D_t$ increases in the available resource $R_t$ and decreases in the terrorist’s unit defense cost $G_t$ (as long as $G_t < R_t/a_t$). The optimal value $D_t$ does not depend on the terrorist’s unit attack cost $B_t$.

Inserting the terrorist’s best response (A1) into (6) yields the government’s first stage utility.
We first determine the interior solution. The government’s first order condition in the first stage implies

\[
\frac{\partial u_i(a_i)}{\partial a_i} = 0 \Rightarrow a_i = \frac{G_i(r_i + b_i c_i)^2}{g_i^3 R_i} \Rightarrow d_i = \frac{1}{b_i} \left( r_i - \frac{G_i(r_i + b_i c_i)^2}{g_i R_i} \right) \tag{A3}
\]

which is inserted into (A1) to yield

\[
D_i = \frac{(r_i + b_i c_i)}{g_i} \left( 1 - \frac{G_i(r_i + b_i c_i)}{g_i R_i} \right), A_i = \frac{(G_i(r_i + b_i c_i) - g_i R_i)^2}{B_i g_i^2 R_i} = R_i \left( 1 - \frac{G_i(r_i + b_i c_i)}{g_i R_i} \right) \tag{A4}
\]

Both players’ choice variables are positive when

\[
R_i \geq \max \left\{ \frac{G_i(r_i + b_i c_i)^2}{g_i r_i}, \frac{G_i(r_i + b_i c_i)^2}{g_i r_i} \right\} \tag{A5}
\]

which means that an interior solution of positive government’s attack and defense and positive terrorist’s attack and defense is guaranteed when the terrorist is sufficiently resourceful.

When (A5) is not satisfied, the government refrains from defending, \( d = 0 \), and focuses on attack, \( a = r_t/g_t \), as determined by (1). Inserting \( \{ d = 0, a = r_t/g_t \} \) into (6) gives

\[
U_i(r_i, g_i, D_i) = \left( \frac{D_i}{D_i + r_i/g_i - G_i} / B_i \right)^m V_i \tag{A6}
\]

Differentiating (A6) for the terrorist gives

\[
\frac{\partial U_i(r_i, g_i, D_i)}{\partial D_i} = 0 \Rightarrow D_i = \sqrt[3]{\frac{R_i}{g_i} - \sqrt[3]{\frac{r_t}{g_i}}} \Rightarrow A_i = G_i \left( \sqrt[3]{\frac{R_i}{g_i}} - \sqrt[3]{\frac{r_t}{g_i}} \right)^2 \tag{A7}
\]

where the latter implication follows from applying (3).
References


